# Are objective chances compatible with determinism? 

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## 1 Introduction

Some aspects of the world seem to be rigidly rule-governed and predictable: deterministic. For example, the orbits of the planets and their moons. Other aspects of reality appear to be a matter of chance: coin flips, dice rolls, the weather. But the world cannot be both chancy and deterministic. Is this tension between determinism and chance illusory? If the tension is real, how do we resolve it? This paper will analyse this tension. There are those who think that chance and determinism really are incompatible; I shall call these people incompatibilists, and the view they espouse incompatibilism. Those who think that the tension can be resolved or defused I shall call compatibilists and their position compatibilism. First I shall introduce the basic concepts of chance and of determinism, then I shall provide an illustration of how incompatibilists can argue that there is an inconsistency in holding that the world is both chancy and deterministic. I shall finish by outlining some responses to the incompatibilist argument.

## 2 What are chances?

We are interested in the relationship between chance and determinism, and we thus need to get clear on what chances are, and what determinism is. This section and the next
will provide this background. The aim of the current section is to describe what a chance is.

We need to know what chances are in order to see whether they conflict with determinism in the ways suggested in the literature. What are chances for? What role does something have to play to be considered as a chance? In what follows, we will understand " $\operatorname{ch}(X)$ " as giving some sort of numerical description of the chance of $X$. So " $\operatorname{ch}(X)=\frac{1}{2}$ " will be understood as saying that the chance of $X$ is a half. We want to find out what it takes to play the chance role. I don't take this to be an exhaustive survey, nor to be providing necessary and sufficient conditions for playing the chance role.

Lewis (1986b) thinks that everything we know about chance is captured by its role in constraining credence, a principle Lewis terms "the Principal Principle". That is, knowing that the chance of $X$ is $\frac{1}{2}$ just means (ceteris paribus) that you believe (or ought to believe) $X$ to degree $\frac{1}{2}$. Even if we don't agree with Lewis' position, we should accept that part of the role of chances is certainly to constrain credence. There is a large and rich literature on what precise form the norm that connects your credences to the known chances ought to take, but that such a connection should be in place is never in doubt. So, that chances constrain credences is certainly at least part of what it is to be a chance.

I am understanding chances as being an objective feature of the world. That is, for the purposes of this paper, I am denying the possibility that chance talk is just elliptical for credence talk. When I say "the chance of $X$ is $x$ " I am making a claim about the feature of the world $X$. A chance is a quantity that attaches itself to an event, in the same way that length is a quantity that attaches itself to a rigid body. That is, chance is a property of events that admits of degrees. There can be more or less chance of some event happening, just like there can be more or less length of an object. Sober (2010) also suggests this analogy.

An important thing to know about a property or quantity is how to recognize when something has it, or to measure how much of it something has. We have various robust
ways for measuring length and weight, say, and that's how we know what we're talking about when we talk about those quantities. For the case of chances, we have no such direct measurements available. We do, however, have some slightly more indirect ways of getting a handle on how much chance certain kinds of events have. We have this in virtue of the link between chances and observed frequencies. It hardly seems worth saying that the observed relative frequency of a coin's landing heads is part of the evidence we have for the chance of heads of that coin (Eagle, 2013). And likewise, it hardly seems worth pointing out that the coin's having a particular chance of heads is what explains that particular frequency. But these two ideas are an important part of the concept of chance.

If we think of logic as assigning 0 to falsehoods and 1 to the truths, then chances seem like an extension of this idea: certain kinds of propositions get intermediate values. The debate about the possibility of deterministic chances turns on whether there can be non-trivial deterministic chances. It is accepted that trivial chance functions - that assign only zeroes and ones - do apply to deterministic worlds. The understanding is that trivial chance functions just encode what things are true and what are false of the world. So the trivial chance functions must be bound by the same laws of logic as apply in the world. This insight will give us a way to impute a structure to chances.

Quantum mechanics contains chance-like objects, ${ }^{1}$
and these are what prompted Popper (1959) to develop his "Propensity theory". Propensities can be understood as a kind of chance. Ismael (2009) argues that chancelike entities are indispensable for physical theories. In any case, chances appear in physical theories: physical theories can give us evidence about the nature and structure of chances. Chances can also appear in non-fundamental sciences: Darwin's appeal to chance variation and Mendel's laws of segregation and independent assortment, for ex-

[^0]ample. ${ }^{2}$ Chances are supposed to be real features of the world just like forces are. A related principle is what Schaffer (2007) calls the "Lawful Magnitude Principle". This says that whatever the true laws of nature are, they determine what the chances are. We can see how our best guesses at the laws of nature - contained in our scientific theories - should give us evidence about chances if this principle held true.

One important role that chances play in physical theories (as well as in other arenas) is that they are part of particular kinds of explanations. (Emery, 2015; Lyon, 2011). ${ }^{3}$ That is, chances explain not just actual observed frequencies, but also more qualitative claims like "it is likely, but not certain, that a silver atom deflected up by this SternGerlach device, passing through a second such device in a slightly different orientation will be deflected up". ${ }^{4}$

As well as relating to logic and truth, chances also connect with certain modal notions like possibility. For example, Eagle (2013) suggests that Leibniz considered chance to be a kind of graded possibility. A sufficient condition for something to be possible is for it to have non-zero chance (Bigelow et al., 1993; Schaffer, 2007). One might want to consider this a necessary condition of possibility; Hájek (ms) appeals to this understanding of the relationship. The claim that chance relates to possibility is currently underdetermined. There are many kinds of possibility one might think are tied to chance: physical possibility?; metaphysical possibility? So there are in fact many different possible properties of chance that differ in what sort of possibility is tied to non-zero chances, and whether the condition is a necessary or a sufficient one. Eagle (2011) ties chances not to the standard modality of possibility, but to what he calls "CAN-claims". In Eagle's view, "CAN $\varphi$ " is a kind of relative modality: relative, that is, to certain contextual features. Eagle argues that chances are also thus context sensitive. This does not mean that they are subjective, however.

I wish to emphasize that even though "CAN" is a relative modality, this

[^1]does not mean that it is in any way epistemic or "subjective"... [T]here is an objective fact of the matter concerning whether a certain contextual restriction is, or is not, in place with respect to a given claim "CAN $\varphi$ "; and an objective fact concerning whether that restriction is, or is not, compatible with the proposition " $\varphi$ ". (Eagle, 2011, p. 284)

In any case, it certainly seems that chance claims are related to claims about some sort of modality of possibility or ability.

Is there a single chance concept? Probably not. But the chance concepts are related: I think that there is enough of a common core of shared properties among the various chance concepts that I can discuss all of them together. ${ }^{5}$ I claim there is some pretheoretic concept of chance. This is, after all, what probability theory was invented to deal with. I think the current technical usage of the term is connected to this pretheoretic usage in the same way that "force" in physics or "continuity" in mathematics are connected to folk uses of those terms. ${ }^{6}$ The above has been my attempt to put some meat on the bones of what we are getting at when we call something a chance: the properties that I take to characterise what chances are. There are some further putative properties of chance that I think serve to distinguish different varieties of chance.

On at least some understandings of chance, there is a connection to the passing of time. The idea is that if we take for granted that events in the past have fixed truth values, then these must be reflected in the chance function which must then assign them trivial chances. Schaffer (2007) takes this to be part of the chance role. On some understandings of chance it makes sense, on others it needn't be part of the role chance plays. This property arguably conflicts with the above condition relating to possibility, if we think of claims like "I could have had toast for breakfast". Given that this is referring to an event in the past, it should have trivial (indeed, zero) chance; but given that it's referring to an event that is possible (on at least some understandings of possibility) then it should have non-zero chance. ${ }^{7}$ (Hoefer, 2007, pp. 554-5) argues that this "Futurity"

[^2]condition is no part of chance.
Some have said that chances are also supposed to be somehow intrinsic. "The intuitive rationale for [Intrinsicness] is that if you repeat an experiment, the chances should stay the same." (Schaffer, 2007, p. 125). We need to be careful about what we mean by the terms we use here. We need to be especially careful about what it is that we duplicate. Consider, first, an example from Gillies (2000, p. 812):

Suppose first we had a coin biased in favour of "heads". If we tossed it in a lower gravitational field (say on the Moon), the bias would very likely have less effect and [ch(Heads)] would assume a lower value. This shows an analogy between probability and weight. We normally consider weight loosely as a property of a body whereas in reality it is a relational property of the body with respect to a gravitational field.

Instead of having the same coin on earth and on the moon, imagine that the coin is on earth, and there is a duplicate of it on the moon. This duplicate has all the same intrinsic properties as the original. This duplicate wouldn't have the same chances, since it wouldn't fall in the same way. So duplicating the chances would mean duplicating not just the coin, but the whole chance set-up. In this case that means duplicating the gravitational field the original (earth-bound) coin was operating in.

Let's return to Gillies' analogy between chance and weight. There are a couple of ways one could read this analogy. One can think of weight as a relational property of a body with respect to a gravitational field, or one can hold fixed the gravitational field, and with respect to that fixed background, weight is an intrinsic property of bodies. Likewise, it's often useful to hold fixed the chance-set up background and think about chances as intrinsic properties. Perhaps this is what Eagle has in mind when talking about how chances are inherently contextual. Another thing to note about the analogy is that weight is connected to the genuinely intrinsic concept of mass. Is there an analogous genuinely intrinsic property that objects have that is to chance as mass is to weight?

[^3]Chances are relational, and thus when we are duplicating events, we need to duplicate the appropriate background conditions too. That is, when we are thinking of intrinsic duplicates, we are thinking of intrinsic duplicates of the chance set up, not of the coins or dice that we colloquially attribute the chances to. Ismael (1996) argues that the intrinsicness of chances, even so construed, is incompatible with a certain kind of view of chance. Roughly, the views of chance that are incompatible with intrinsicness are those that make chance relative to a reference class. ${ }^{8}$ This is no problem for Schaffer since he is interested in a specific understanding of chance. ${ }^{9}$ I don't want to commit myself to such an understanding - I am trying to remain theory-neutral - so I cannot endorse Intrinsicness as part of the chance concept. It is part of what differentiates different kinds of chance concepts.

Chances are sometimes thought of as (part of) what causes the outcomes of chance set-ups. In different ways Schaffer (2007) and Lyon (2011) are thinking in this way.

One final alleged property of chances is that they obey the axioms of probability theory. This is an assumption that Bradley (2016) questions. Very little of my discussion will depend on making any such assumption.

Let's summarize the above discussion as follows:

Credence Chances are what constrain credences through the principal principle
World Chance facts are claims about the world
Frequencies are evidence Observed frequencies are evidence of chances
Chances explain frequencies How the chances are should explain the frequencies we observe

Logic Chances relate to logic and truth
Theories Scientific theories tell us about the chances
Lawhood Chances are determined by laws of nature
Explanation Chances explain some features of the outcomes

[^4]Possibility $X$ is possible if (and only if?) $\boldsymbol{c h}(X)>0$
Futurity If $X$ is an event in the past, its chance is 1 or 0
Intrinsicness If $X^{\prime}$ is an exact duplicate of $X$, then $\boldsymbol{\operatorname { c h }}(X)=\boldsymbol{\operatorname { c h }}\left(X^{\prime}\right)$
Causation Chances cause the outcomes of chance set-ups
Chance-probabilism Chances obey the axioms of probability theory

I don't take this to be an exhaustive list, but I hope that the above properties serve to triangulate the concept of interest.

So we have now seen the sorts of properties chances are supposed to have, and how they are supposed to connect to other theoretically valuable concepts like causation, credence, possibility and so on. Let's now turn to how we might distinguish different kinds of chance concepts. ${ }^{10}$

One core disagreement on the nature of chance is whether chances are taken to be causal and dispositional or whether they are a relational property of some sort. ${ }^{11}$ That is, some have a metaphysically "thick" notion of chance: it is the chance that (sometimes?) causes the chancy outcome that actually eventuates. ${ }^{12}$ Others - most notably Lewis take chances to be relational properties. The chances attached to a flip of a coin are determined not just by the intrinsic properties of the coin toss, but of the outcomes of past and future similar coin tosses, and by what the best theory of coin tosses says the chances are. For Lewis (1986b), chances supervene on the nonmodal properties of spacetime points. As Hoefer (2007) puts it, "chances are constituted by the existence of patterns in the mosaic of events in the world". ${ }^{13}$

[^5]Returning to which properties of chance might distinguish different kinds of chance, properties like Futurity and Intrinsicness seem like the kinds of things that are part of dispositionalist chance concepts, but not of relationalist chance concepts. Hoefer and Ismael (who deny the former and latter of these respectively) are both arguing from a broadly Humean (i.e. relationalist) understanding of chances.

Another difference worth drawing attention to is the diversity in answers to the question: "what things have chances attached to them?" Some take it that chances attach to objects: the coin has chances. Some instead emphasise that it is the chance set-up that has chances: the coin in its particular gravitational field, atmospheric conditions, the way it was flipped and so on. Yet others focus on the events that result from a particular set up as the things that have chances: it is the coin's landing heads that has a chance. Suárez (2013) puts Karl Popper in the set-up camp, and C.S. Pierce in the object camp. These are different ways of talking, but it isn't clear to me that such differences in phrasing reflect a difference in ontology. Or rather, the difference in ontology hinted at by these different ways of speaking is just the dispositionalist/relationalist distinction again.

A third distinction is that between "single-case" and "long-run" chances: does a coin flip have chances just in virtue of being a coin flip, or does it have chances only in virtue of its belonging to some long sequence of coin flips? Ismael (2011) makes the distinction between "single case chance" and "general chance" ${ }^{14}$ the former being attached to oneoff events, the latter to sequences of repeatable events. Do chance set-up tokens have chances? Or are chances something that only belong to types of chance set-up? Suárez (2013) puts both Popper and Pierce on the side of type chances, while he himself opts for a view where tokens have chances.

One could be a realist about the theoretical terms - like "chance" - that one posits, or one could take a "hypothetical" view. Popper was a realist about chances: he thought that claims about chances could be falsified through actual data (presumably data about frequencies). Pierce's hypothetical, pragmatist view took chances to be measurable - if

[^6]at all - only indirectly. The uses of chance talk were in explanations, not in predictions. Their worth was in their usefulness as part of our explanatory toolkit.

Chance is among a bundle of concepts that are often used interchangeably but should perhaps be kept separate. This bundle includes the concepts "chance", "indeterminism", "unpredictability" and "randomness". Earman (1986), von Plato (1982) and Eagle $(2013,2005)$ contribute to separating out these similar ideas. Norton (2008) can also be read as contributing to this project.

This section has given a (partial) picture of the chance concept or concepts I am focusing on. I am going to talk as if there is a single concept for reasons of grammatical simplicity, but I don't expect anything I have to say to rely on there being a unified concept. The plan is to try, as much as possible, to be "theory neutral". This means that I want to argue in such a way that whichever view of chance you have you can see the force of the arguments outlined below. The important thing is that chances are a feature of the world, and they have some relation to credence, frequency, logic and scientific theories.

Of course, not all ordinary language uses of "chance" are going to be amenable to this analysis. For example, when I say "I had a chance to go swimming with dolphins", this should be understood as referring to some sort of opportunity, rather than to some dispositional property with a (possibly) probabilistic structure. Such uses of the word "chance" aren't connected to frequencies or to scientific theories in the way we consider our chance concept to be.

## 3 What is determinism?

We now have an understanding of the things someone could mean when they talk about chance. Now we need to do the same exercise for the concept of "determinism". Thankfully, there isn't quite so much ground to cover here: most people pretty much agree about what it is for a world to be deterministic. Let's start with John Earman's characterisation of determinism (Earman, 1986). For a possible world $w, W_{w}$ is the set of physically possible worlds: worlds that have the same laws of nature as $w$. A possible
world $w$ is Laplacian deterministic if: for all $w^{\prime} \in W_{w}$, if $w$ and $w^{\prime}$ agree on all occurrent facts at time $t$, they agree on all occurrent facts at other times. ${ }^{15}$ The occurent facts are those things true of the world at that time. The qualification is needed to rule out trivialisations of the concept of determinism by dirty tricks such as contemplating facts like "It will be the case that it rains tomorrow". Note that what is deterministic in Earman's case is a world. Earman's discussion is put in terms of possible worlds, but this does not signal any commitment to modal realism. It just affords a neat way to describe how it is that the concept of determinism is connected to laws of nature.

Earman then offers a more general version of determinism. A world $w$ is $\left(R_{1}, R_{2}\right)$ deterministic if, for all $w^{\prime} \in W_{w}$, if $w$ and $w^{\prime}$ agree on $R_{1}$, then they agree on $R_{2}$. Laplacian determinism can be recovered by understanding $R_{1}$ as the occurrent facts at a time $t$, and $R_{2}$ as the whole of spacetime. So $R_{1}$ and $R_{2}$ are collections of occurrent facts. Earman actually required that $R_{1}$ and $R_{2}$ be (all the occurrent facts about) regions of spacetime, but I don't see why we can't define kinds of determinism where the sets $R_{i}$ are other kinds of sets of facts.

So, to work out whether some world is deterministic in Earman's sense, we need to know what the laws of nature are; that is, what $W_{w}$ is. And we need to know what count as the occurent facts at a time.

Werndl $(2009 \mathrm{a}, 2011)$ characterises determinism in the language of phase spaces. A description is deterministic if, for a given lapse of time $\delta t$, there is a function that takes the state at $t$ and outputs the state at $t+\delta t$. A description of a system involves a collection of states, a measure on the (sigma-algebra over the) set of states, and a flow $\left\{\Phi_{t}\right\}$ : a parametrised family of functions that compute the changes in state. These functions must be such that $\Phi_{s}\left(\Phi_{t}(x)\right)=\Phi_{t+s}(x)$. Putting Werndl's theory in Earman's terms, if descriptions $w$ and $w^{\prime}$ agree on the state at $t$ and the flow, then they agree on the state at other times. Despite their apparent differences, Werndl's and Earman's characterisations of determinism are closer than they seem. Recall that for Earman, the

[^7]set of worlds $W_{w}$ is a set of nomically possible worlds. That is, worlds that share the laws of $w$. Arguably the flow just is the characterisation of the laws of nature, so all $w^{\prime} \in W_{w}$ share the same flow. So if $w^{\prime}$ is nomically possible (with respect to $w$ ) and they agree on the state at $t$, then they agree also on the flow by definition, and thus the two characterisations of determinism are very similar. Note that the identity of the flow in two worlds is a stronger property than one world's being nomically possible with respect to another. It entails a stronger logic of nomic possibility. ${ }^{16}$

List and Pivato (2015) define determinism as the uniqueness of the continuations for a truncated history. This amounts to Earman style $\left(R_{1}, R_{2}\right)$-determinism where $R_{1}$ is (the occurrent facts about) up to some point in time, and $R_{2}$ is (the occurrent facts about) the future. What is determining which continuations of a truncated history are permissible are the standards of nomic possibility. So again, List and Pivato's analysis of determinism, apparent differences notwithstanding, is very close to Earman's.

The thing to emphasise at this point is that determinism is intimately tied to conceptions of nomic possibility, and thus to the concept of a law of nature.

## 4 Determinism, possibility and incompatibility

We are now in a position to discuss more carefully the alleged incompatibility between (non-trivial) chance and determinism. Consider a world $w$ and a time $t$ that is such that there is a nontrivial chance that $\phi$ will occur, where $\phi$ is some future contingent ("future" with respect to $t$ ). Given the connection between chance and possibility, this means that it is (nomically) possible that $\phi$, and also it is (nomically) possible that $\neg \phi .{ }^{17}$ Say, for concreteness, that as a matter of fact $\phi$ turns out to be true. But there is a world $w^{\prime}$ that agrees with $w$ on all the occurrent facts at $t$, including the chance facts, but is such

[^8]that $\neg \phi$ turns out true at $w^{\prime}$ : that's what is required in order for $\neg \phi$ to be nomically possible. $w$ and $w^{\prime}$ agree on all occurent facts at $t$, and on the laws, and yet there is a fact at a later time (namely $\phi$ ) about which they disagree. Thus $w$ is indeterministic. So it appears that there is something of a tension between (non-trivial) chances and determinism. ${ }^{18}$ If we endorse the claim that $\phi$ is possible only if $\phi$ has non-zero chance, then we also have the converse problem. Even if we don't endorse this claim, the worlds that are indeterministic but have only trivial chances are somewhat pathological.

## 5 The method of arbitrary functions

One approach to resolving the tension between chance and determinism starts by noting a certain kind of mathematical result. Sometimes, when you have a system where the final state is determined by a well-behaved function of the initial state, then the distribution of final states is independent (or almost independent) of the distribution of initial states. Often, deterministic dynamics are described by such a function (consider Werndl's characterisation of determinism for a fixed time interval). When this is the case, that (somewhat) independent final distribution could be considered a kind of objective chance distribution. The basic idea of the approach - called the "method of arbitrary functions" - goes back to Henri Poincaré (1895), and is mostly carefully and extensively worked out by Michael Strevens (2003; 2013).

How might such an "initial distribution-independent" final distribution occur? Imagine a case where small differences in initial state can yield different final states. For example, a coin tossed with slightly different angular momentum could land heads rather than tails. Call the preimage of heads the set of initial states that yield heads when that state is subjected to the function that characterises the dynamics. If the preimages of the various outcomes are sufficiently mixed up together, then any sufficiently smooth distribution over initial states will yield roughly the same distribution over final states.

[^9]Such distributions, argues Strevens, are good candidates to be the objective chances. We don't really have time to discuss this proposal in detail, but let's note two things: first, in a deterministic world, there is a specific initial condition with a specific outcome under the dynamics; second, the "mixed-up-ness" of the preimages depends on a choice of outcomes (heads and tails, rather than a more fine-grained space of outcomes).

## 6 The Humean chance solution

Another approach to resolving the tension between chance and determinism starts by arguing for a particular view of what the actual laws of nature are like (Albert, 2001; Loewer, 2001, 2004). The claim is that the best system consists in the deterministic dynamics of Newtonian mechanics, the postulate that the inital condition of the universe was a low-entropy state and a postulate about the probability distribution of initial conditions. This system is simple, and yet extremely informative. And, importantly, it gives non-trivial chances to particular events. So: the laws are thus and so, and they entail these non-trivial probabilities. These probabilities are dervied from the laws of nature in such a way as to qualify them as chances. Thus, chance and determinism are compatible. Arguably, this solution depends on a Humean relationalist view of chance, and is rather unfriendly to a dispositionalist view.

Another Humean-based theory of chance is that discussed in Frigg and Hoefer (2010). What they do is define what it is to be a "Humean Objective Chance" (HOC), and then show that such things are compatible with a deterministic world. An HOC is a function that satisfies the axioms of probability, plays the right role with respect to chance-credence coordination, and supervenes on the basic facts (the Humean mosaic) "in the right way". ${ }^{19}$ Frigg and Hoefer argue that orthodox "chancy" set-ups like coin tosses satisfy their definition and are thus chances. Such set-ups are consistent with a deterministic micro-physics, thus compatibilism is correct, according to Frigg and Hoefer. How exactly have these solutions side-stepped the problem we highlighted at the end of

[^10]the last section? Recall that we highlighted the tension between chance and determinism by focussing on the concept of (nomic) possibility. Consider two distinct worlds that are drawn from the same distribution of (low entropy) initial conditions. Is one nomically possible with respect to the other? If one says yes, then the dynamics of these worlds aren't really deterministic (since they will not agree on all occurent facts). If one says no - these initial conditions are not nomically compossible - then it doesn't seem as if one really has non-trivial chances. Eric Winsberg puts the problem like this:

The fundamental problem with understanding the probabilities in [Statistical Mechanics] to be objective is that we are meant to posit a probability distribution over a set of possible initial states, while we suppose, at the same time, that in fact only one of these initial states actually obtained. (Winsberg, 2008, p. 873)

Or take the Frigg and Hoefer version. Consider two (deterministic) worlds that are similar enough in their histories and laws of nature to have the same (or very similar) HOCs. Is the future state of one world nomically possible with respect to the current state of the other? Again, if we answer "yes" then the worlds don't count as deterministic, and if we answer "no", it is unclear that we have genuine chances compatible with possibility in the right way. There seems to be a desire in this literature to have one's cake and eat it too, and I don't see how such a move, on its own, really solves the tension between objective chance and determinism. This isn't supposed to be a knockdown argument against this approach to chance and determinism, but rather a warm-up for another approach to defending compatibilism.

## 7 "Levels" to the rescue?

A deterministic system can appear chancy if the dynamics are chaotic: nearby initial conditions eventually become arbitrarily distant. ${ }^{20}$ And a system that is chancy can still appear deterministic in its coarse grained properties: gas molecules may bounce around

[^11]more or less at random, but the gas will expand to fill a chamber evenly. Can we use this idea of a "coarse-graining", or a "level" to resolve the tension between chance and determinism?

Schaffer (2007) makes a careful case for incompatibilism, and Glynn (2010) rebuts him by bringing in a level-relativity of chances. The goal of the remainder of this paper is to describe, in detail, how this level relativity helps.

Now, let $X_{l}$ be the collection of level $l$ (occurrent) facts at a time, and $Y_{l}$ likewise. For the moment, "level" is just a free parameter in our theorising: we'll discuss what we mean by a level in more detail later. There is no problem with a world $w$ being $\left(X_{l}, Y_{l}\right)$-deterministic while not being $\left(X_{l}, Y_{l^{\prime}}\right)$-deterministic where $l^{\prime}$ is a fine-grained, lower level than $l$. Two worlds can coincide on all the higher-level thermodynamic facts about a gas in a box - pressure, temperature, etc - and yet differ on the lower-level facts about distribution of particular gas molecules.

Let's run through our argument for incompatibilism again, keeping track of the level involved so as to see where it has gone wrong. Recall we had a world $w$ that had a nontrivial chance of $\phi$ and thus a nontrivial chance that $\neg \phi$. We should say that the nontrivial chance of $\phi$ is a chance at some level. It followed from this that $\phi$ was possible, and that $\neg \phi$ was also possible. Let's make possibility level-relative too. Lewis (1986a) has convinced us that possibility is relative to what facts one holds fixed - I can speak Finnish (my vocal cords allow it) but I can't speak Finnish (I haven't learned the language) - it's not much of a stretch to see possibility has relative to what facts are available at a level. So it follows from its nontrivial chance that $\phi$ is possible at whatever level the chance is at. Using the same reasoning as we did above, we can argue that that level is not deterministic. But it does not follow that the world cannot be deterministic at other levels. There doesn't appear to be much we can conclude about other levels above or below $l$ from the fact that $l$ is deterministic or indeterministic (Earman, 1986; Werndl, 2009a, 2011; List and Pivato, 2015).

Let's step back from the levels talk for a minute and think about languages. Different scientists have different vocabularies of scientifically significant predicates. Psychologists
talk about "concepts", and "learning"; Biologists talk about "tigers", and "fitness"; Physicists talk about "electrons", and "fields"; Economists talk about "rational agents", and "money". Many fields appeal to laws of nature described in the language of that level. It is this talk of laws of nature at different levels that Glynn (2010) takes as his starting point. Those with anti-reductionist or emergentist tendencies will accept this talk at face value and see these as legitimate laws. ${ }^{21}$ Given the level relativity of laws, determinism too comes out as a level relative concept. It might be that the laws with respect to some low level language are such that the system appears deterministic, while the high-level laws make it appear that the system is high-level indeterministic or vice versa. Which $w^{\prime}$ are in $W_{w}$ depends on what the laws are: on what worlds are nomically possible.

## 8 Non-fundamental chance?

Some might object that this is doing violence to the concept of determinism. Determinism is surely a property of the system with respect to the lowest-level laws from the perspective of the language that consists in all and only the perfectly natural properties.

Are higher-level chances chances? Schaffer (2007) dismisses them as "epistemic chances", Lyon (2011) wants to insist that they are objective "counterfactual probabilities" but not chances. ${ }^{22}$ Lewis, responding to Levi's discussion of his view, calls them "counterfeit chances". Schaffer suggests that epistemic chances are candidates to play the chance role, but that fundamental-level chances play the role better, and fundamental-level chances in deterministic worlds are trivial. (Glynn (2010) contests this point, as does Emery (2015)). Lyon, drawing on Schaffer, focusses on the credence-guiding role that chances are supposed to play. Lyon argues that since propositions about the history and about the laws are clearly admissible, high-level non-trivial chances in deterministic worlds

[^12]cannot play the credence-guiding role.
Are we having a merely verbal dispute? What's at stake? First, what precisely is the contrast? Are the "real" chances those that occur at the fundamental level when using the perfectly natural predicates? If so, then scepticism about these concepts - the fundamental level, the perfectly natural predicates - is reason to be sceptical that "real" chances exist. Is there such a language? Is there such a level? The burden of proof is on the proponent of such a view to provide positive answers to these questions. van Fraassen (1989) provides some reasons to be sceptical. ${ }^{23}$

## 9 Conclusion

We have seen that the concepts of chance and determinism are not univocal, but that there seems to be something of a tension between core elements of these concepts. We have further seen that there are options for those who wish to defuse this tension.

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[^0]:    ${ }^{1}$ What quantum mechanics tells us about the compatibilism debate is somewhat murky. There are deterministic interpretations of quantum mechanics that seem inimical to chance (e.g. the Everett interpretation) and deterministic interpretations that are perhaps more friendly to chances (e.g. Bohmian mechanics). Then there are explicitly indeterministic interpretations (e.g. GRW theory) that seem unfriendly to determinism. Exploring these topics in more detail would take use too far afield.

[^1]:    ${ }^{2}$ See (Glynn, 2010, Section 3.1) for a discussion of special science chances.
    ${ }^{3}$ Lyon wants to draw a distinction between chance and "counterfactual probability", which seems to track the distinction I will draw later between fundamental chance and high-level chance.
    ${ }^{4}$ I borrow this example from Emery (2015)

[^2]:    ${ }^{5}$ Chance is a quantity and (Ellis, 1966, pp. 34-8) suggests that all quantities are cluster concepts.
    ${ }^{6}$ Thanks to an anonymous referee for suggesting this analogy.
    ${ }^{7}$ One explanation of the above is that we are equivocating between different senses of possibility. Recall

[^3]:    the earlier comments about the underdetermination of the relationship between chance and possibility.

[^4]:    ${ }^{8}$ Ismael has Lewis in mind, here. I'll discuss different views of chance, including Lewis' in a moment.
    ${ }^{9}$ Schaffer is committed to a dispositional reading of chance. See later in this section.

[^5]:    ${ }^{10}$ This section has been strongly influenced by the distinctions Suárez (2013) draws between Karl Popper's and C.S. Pierce's views on chances. See also Suárez (ming)
    ${ }^{11}$ While Suárez (2013) makes a distinction between "causal" and "relational" views, his distinction does not match mine. Indeed, his causal/relational distinction maps more closely to the distinction I refer to by the labels "object chance" and "set-up chance" below.
    ${ }^{12}$ For a recent overview of so-called "propensity theories", see Berkovitz (2015).
    ${ }^{13}$ The Stanford Encyclopedia of Philosophy has articles on David Lewis and on David Lewis' Metaphysics, both of which have sections on "Humean supervenience", and the article on Interpretations of Probability has a section on "Best system interpretations"; all of which provide good starting

[^6]:    points for this topic
    ${ }^{14}$ Although she uses the term "probability" where she means "chance".

[^7]:    ${ }^{15}$ If we were interested in relativistic spacetimes, we could replace talk of "time" with "timeslice" throughout.

[^8]:    ${ }^{16}$ Earman explicitly avoids committing himself to such a stronger - S 5 - modal logic of nomic possibility. Bigelow et al. (1993) explicitly endorse this stronger logic of possibility.
    ${ }^{17}$ Note that we are here relying on the fact that if $0<\boldsymbol{\operatorname { c h }}(\phi)<1$ then $0<\boldsymbol{\operatorname { c h }}(\neg \phi)<1$. This follows from the axioms of probability theory, but even if we didn't want to endorse chance-probabilism, it seems plausible that at least some instances of non-trivial chances would be such that the above holds.

[^9]:    ${ }^{18}$ This argument is similar to one argument offered by Schaffer (2007): namely that nontrivial chances in deterministic worlds cannot have the right connection to Possibility. Schaffer offers more discussion of this example, and several other arguments besides.

[^10]:    ${ }^{19}$ Frigg and Hoefer actually make the HOC a number such that the number is determined by a function with the properties listed here, but I don't think that distinction really matters.

[^11]:    ${ }^{20}$ Defining precisely what "chaos" is is notoriously tricky: Smith (2007, 1998); Werndl (2009b).

[^12]:    ${ }^{21}$ See, for example, Cartwright (1999) or Dupré (1993).
    ${ }^{22}$ In fact, I think Lyon would prefer if we just got rid of the term "chance". But Lyon still wants to draw a distinction between the fundamental chances and the counterfactual probabilities; a distinction I don't find compelling.

[^13]:    ${ }^{23}$ What van Fraassen actually provides is reason to be sceptical that any term of current science, or indeed any term of any possible future science, is a perfectly natural predicate.

